



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

If we take $80p=1248$ or $p=15+3/5$, we find $h=-(4+2/15)$, which gives $a^2=(17/15)^2$. We also have for the three sides, after some easy reductions: 510, 466, 884, and for the medians 659, 683, 208. This is perhaps the simplest case in whole numbers.

43. Proposed by H. C. WILKES, Skull Run, West Virginia.

To find, if possible, a right angled triangle, the bisectors of the acute angles of which, can be expressed by integral whole numbers.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let ABC be a right triangle, right angled at C , AD the bisector of $\angle A$, and BE the bisector of $\angle B$.

Put $BC=a$, $AC=b$, $AB=c$, $DC=c_1$, $EC=b_1$, $EB=c_2$, and $AD=c_3$. Then $BD=a-c_1$, and $AE=b-b_1$. From geometrical relations we obtain $a^2+b^2=c^2 \dots \dots (1)$; $c_1^2=ac-b_1(b-b_1) \dots \dots (2)$; $b-b_1 : b_1 = c : a \dots \dots (3)$.

From (3) we get $b : b_1 = c+a : a$; whence $b_1 = ab/(c+a)$, and $b-b_1 = bc/(c+a)$.

$$\therefore c_1^2 = ac - ab^2/c(c+a)^2 = ac - ac(c^2-a^2)/(c+a)^2 = 2a^2c/(c+a).$$

By a similar process, we find $c_2^2 = 2b^2c/(c+b)$.

From (1), $c^2-b^2=a^2$, or $(c+b)(c-b)=a^2$. Put $c+b=tp^2$ and $c-b=tq^2$. t , p , and q being any values. Then $a=tqp$, $b=t(p^2-q^2)/2$, and $c=t(p^2+q^2)/2$. Whence $c_1^2=2t^2p^2q^2(p^2+q^2)/(p+q)^2$, and $c_2^2=t^2(p^2-q^2)^2(p^2+q^2)/4p^2$.

When $p^2+q^2=\square$, $c_2^2=\square$, and $c_1^2=2\times\square$. When $p^2+q^2=2\times\square$, $c_2^2=2\times\square$, and $c_1^2=\square$.

\therefore Both bisectors cannot be rational; one of them will be $\sqrt{2}$ times a number, when the other is a rational whole number.

II. Solution by the PROPOSER.

Let bx , by , and $x+y$ be, respectively, the sides and base of a right angled triangle, and let x and y be the greater and less segments of the base cut by the bisector. Then the bisector will be $\sqrt{y^2(b^2+1)}$ and if the bisector be integral, b^2+1 must $=\square$. b must therefore be an improper fraction, and will always be the quotient of the sum of the other two sides divided by the bisected side.

Now let CAB be a triangle, and let $AB=x^2+y^2$, $CA=x^2-y^2$ and $CB=2xy$, $CA+AB/CB=x/y$. $(x/y)^2+1$ may be a square, but $AB+CB/CA=[(x+y)/(x-y)]^2+1$ will be a multiple of the $\sqrt{2}$ and cannot be a square.

\therefore If a rational right angled triangle have an integral bisector of one of its acute angles, the bisector of the other acute angle must be a multiple of $\sqrt{2}$ and cannot be integral.

[Remark.—On page 155, Vol. II. of the MONTHLY, we have, when the sides are 59.4107, 47.4072, 35.8067, the bisectors 40 and 50. It is doubtful whether the sides and bisectors both can be integral. ZERR.]

